

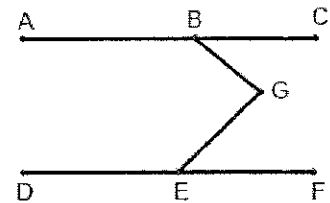
2013 John O'Bryan Mathematical Competition
Freshman-Sophomore Individual Test

Directions: Please answer all questions on the answer sheet provided. All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value.

1. Find the value of x that satisfies $\frac{x-4}{x-5} = \frac{x+3}{x+8}$. Give your answer as an **improper reduced fraction**.

2. A circle with center $(5, -7)$ is tangent to the x -axis and has area $k\pi$. Find the value of k .

3. In the diagram at the right, the line containing A, B, and C is parallel to the line containing D, E, and F. If $\angle BGE = 88^\circ$ and $\angle FEG \cong \angle CBG$, find the degree measure of $\angle ABG$.



4. A regular polygon has k sides. Half the measure of an interior angle is 6 more than three times the measure of an exterior angle. Find k .

5. The largest number which can be obtained as the product of two positive integers whose sum is 104 can be expressed as $a^b c^d$ where $a, b, c,$ and d are positive integers and where a and c are prime numbers such that $a < c$. Find the value of the expression $(ac - bd)$.

6. When $x = -9$, x^2 exceeds $-x^2$ by qx . Find the value of q .

7. In the addition alphametic shown to the right, each letter stands for the same digit throughout the puzzle. No digit is represented by more than one letter. For your answer write the 4 digits (from left to right in order) that represent ROOM.

$$\begin{array}{r} \\ \\ \\ \hline \\ \\ \\ \end{array}$$

8. Find the perimeter of a square inscribed in a circle of area 81π . Give your answer in the form $c\sqrt{d}$ where c and d are integers.

9. Think of the hands of a clock as lying along two radii of a circle. Find the degree measure of the minor arc intercepted by the hands of a clock at 5:50 AM.

10. In a survey of 100 people, it was found that 64 people read the most recent issue of *Newsweek* and that 75 people read the most recent issue of *Sports Illustrated*. It was also found that eight of those surveyed read neither of these issues. If one of these 100 people is selected at random, find the probability that the person selected read both issues. Express your answer as a **decimal**.

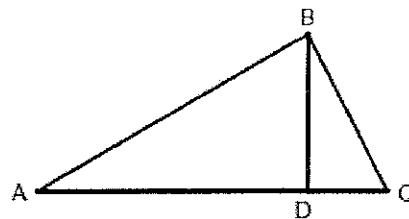
11. Find the sum of all distinct positive integral factors of 2013.

12. For all real numbers, define $a \otimes b = \frac{a^2 + 3b}{4a}$. If $a \otimes 17 = 5$, find the sum of all possible value(s) of a .

13. In the diagram $\angle ABC = 90^\circ$ and D lies on \overline{AC} such that $\overline{AC} \perp \overline{BD}$.

Suppose also $AC = 17/15$ while $AB = 1$. Let $\frac{AD}{DC} = \frac{m}{n}$ where m and

n are relatively prime positive integers. Find the smallest possible value of the expression $(2m + 3n)$.



14. The sides of $\triangle ABC$ have lengths 1, 2, and $\sqrt{3}$. The sides of $\triangle DEF$ have lengths 2, 2, and $\sqrt{8}$. If one of the six angles A, B, C, D, E, or F is selected at random, find the probability that the degree measure of the angle is a multiple of 30° . Express your answer as a common fraction reduced to lowest terms.

15. The hypotenuses of two right triangles have equal lengths. All legs of these triangles have lengths that are integers. One of the triangles has congruent legs while in the second, one leg is 14 units longer than the other. Find the smallest possible sum of the areas of these triangles.

16. Two concentric circles have diameters that sum to 100. \overline{AE} is a chord of the larger circle and has length 40. Furthermore, A, B, C, D, and E lie on \overline{AE} in that order and divide \overline{AE} into equal lengths. The smaller circle passes through B and D. The area of the smaller circle is $k\pi$. Find the value of k .

17. Bubbles can completely fill her empty bathtub in 40 minutes with the hot water faucet alone. The cold water faucet alone will completely fill the empty tub in 30 minutes. A completely full tub will drain in 20 minutes. Bubbles turned on both water faucets to fill her empty tub, but forgot to close the drain. After 8 minutes she realized the error and closed the drain. After another 7 minutes Bubbles turned off the cold water faucet so that the water would be warm. In total, how many minutes did it take Bubbles to completely fill her bathtub?

18. The roots for x of the quadratic equation $x^2 - kx + w = 0$ are unequal, and both roots are positive even integers. Find the smallest possible value of w .

19. Solve for x if $\frac{2}{x^2 + 9x + 20} + \frac{2}{x^2 + x - 12} = \frac{5}{x^2 + 2x - 15}$.

20. An isosceles trapezoid with legs of length 10 has bases of lengths 24 and 40. Find the length of an altitude to a base of this trapezoid.

Name: _____ **ANSWERS** _____

Team Code: _____

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1. $17/6$ Must be in
this form

11. 2976

2. 49

12. 20

3. 136 Degrees (°)
Optional

13. 642

4. 15

14. $2/3$ Must be
reduced
fraction

5. 18

15. 576

6. -18

16. 484

7. 1009

17. 36 Minutes
Optional

8. $36\sqrt{2}$ Must take
this form

18. 8

9. 125

19. -16

10. 0.47 or $.47$ Decimal point
must be clear

20. 6

Awards Lists and Solutions to the Team Competition may be found at
<http://www.nku.edu/~math/job/>